Graphical User Interface for pricing Cryptocurrency Options under the Stochastic Volatility with Correlated Jumps model

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Prof. Dr. Wolfgang Härdle

Prof. Dr. Cathy Y. Chen

School of Business and Economics

Ladislaus von Bortkiewicz Chair of Statistics

Humboldt Universiät zu Berlin

by

Ivan Perez

586281

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Abstract

Since their creation, cryptocurrencies have attracted the attention of many people, both in academia and in industry, not only because of their innovative technology but also because they have become important financial assets. However, the derivatives market has not yet been developed, making it difficult to better manage the risk associated with the high volatility presented by some cryptocurrencies. Understanding the prices of cryptocurrencies is an important aspect in the development of a contingent claims market, specially when there are no fundamentals at hand, as is the case of stocks, for example. This thesis aims to help to fill some spaces through a Shiny application with three purposes: to use the SVCJ (Stochastic Volatility with correlated Jumps) model to estimate the returns of the cryptocurrencies that are part of the CRIX Index, this in order to bring closer practitioners to the, not that widely known, SVCJ model. The second objective is to extend the econometric analysis to some cryptocurrencies other than Bitcoin. The third objective is to use the econometric results to do an option pricing exercise, easily accessible trough the aforementioned app and thus providing a small grain of sand for the development of a derivatives market for cryptocurrencies

Key Words: CRIX, Bitcoin, Cryptocurrency, Option Pricing, Risk Neutral Density, SVCJ

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1 Introduction

The first time the world heard about the word Bitcoin was in the year 2008 after Satochi Nakamoto published the famous White Paper (Nakamoto, 2008). Bitcoin emerged as the first of its kind into what today we know as cryptocurrencies, henceforth cryptos. Some common characteristics of cryptos are still revolutionary, such as, the use of immutable databases, sophisticated cryptography techniques and distributed ledger technologies, all of them represented in what we know as the blockchains. A concise and elucidating explanation of the mechanics of cryptocurrencies can be found in (Härdle et al., 2018).

Some other characteristics of the cryptos are not free from controversy, such as the absence of a Central Bank to support the value of the currency, or the possible presence of price bubbles (Cheah and Fry, 2015), or the misuse of some Initial Coin Offerings ICO (see bitcointalk.org), or even the high concentration of some wallets, see (Amoros, 2016). Independent of the opinion everyone has, one important fact is that as of October 14th, 2018 there are almost 2.080 cryptos with a total market capitalization of 201 billion USD (according to coinmarketcap.com) which, under any perspective, is a considerable amount (on January 7th, 2018 a maximum market capitalization of 813 billion USD was achieved)

Another important aspect of the cryptos so far is the lack of some legal and normative controls, making it difficult for market participants to know their rights or to have an adequate control of the risks when investing in cryptos, see (Girasa, 2018) and also (Ryznar, 2018). Usual authorities, such as the Securities and Exchange Comission (SEC), have been issuing bulletins to inform investors about potential risk in the crypto market (see for example (SEC, 2014) and (SEC, 2017)), without building so far a complete legal frame, see (Crabb, 2017).

On the other hand, the initial role of central and commercial banks has been dedicated to monitor, in a passive way, the evolution of the crypto-market. However, that has starting to change until reaching the point that some central banks are considering having their own cryptos, (Bech and Garratt, 2017) (see also Bitcoin.com (2018)) or, in the case of commercial banks, dedicating whole business units for the development of blockchain related technologies (e.g. JP Morgan's Blockchain Center of Excellence). Some theoretical basis for the incorporation of cryptos into the monetary policy can also be fount in (Almosova, 2018).

Among this universe of cryptos, start ups, ICOs, bulletins, successes and failures, typical of a developing market, there is one very important aspect that needs to be fully created and formalized if we want cryptos to succeed, that is, a derivatives market for cryptos.

Some characteristics of the cryptos have been documented, which support the previous idea about the necessity of a contingent claim market. Some cryptos, and among them the most analyzed Bitcoin (BTC), are well known for having high volatility and a speculative pattern. For example (Cheah and Fry, 2015) found that Bitcoin contain a considerable speculative component making it susceptible to bubbles. (Ciaian et al., 2016) found that traditional market forces could drive Bitcoin price but they vary over time. In turn, (Kristoufek, 2015) found, after analyzing fundamental, speculative and technical sources, that Bitcoin has properties of both a standard financial asset and a speculative one. The degree of uncertainty has been such that IG Group, the world's largest online trading platform, suspended trading of some of its Bitcoin derivatives on November 2017 after roaring demand for the products left the company facing a high security risk (see Financial Times).

Attempts to create a derivatives market are already on its way. Precisely one of the first was the Commodity Futures Trading Commission (CFTC) approval of LedgerX for clearing derivatives, on July 2017 (see CFTC). LedgerX is an institutional trading and clearing platform focused on trading and clearing swaps and options on digital currencies. It is not a direct market for options whose underlying asset are cryptos, but it is an important approximation to relate cryptos with derivatives.

In the same direction as the previuos, an important milestone was achieved in October 2017 when the CME (Chicago Mercantile Exchange) announced it plans to launch Bitcoin futures, which indeed they did on December 2017. The CME is the worlds leading and most diverse derivatives marketplace. For its part the CBOE (Chicago Board Options Exchange), the largest U.S. options exchange, did the same on December 2017, with a Cboe Bitcoin Future.

1.1 Motivation

One important aspect for any market, specially for the derivatives market, is to understand the asset price formation, and in the case of cryptos there are some particularities, such as the absence of fundamentals that supports the price, compared to some other assets (i.e. fundamentals such as sales, assets, revenue, etc). That is why statistical and mathematical tools that help to understanding crpytos price behavior play an important role.

An initial cross-seection comprehensive analysis of the cryptos was done by (Elendner et al., 2016), where cryptos returns and correlations were analyzed and some portfolios were built. For its part, an initial econometric analysis of the CRIX Index and Bitcoin can be found on (Chen et al., 2016) where after using several models such as ARIMA models and GARCH family models authors found volatility clustering phenomenon and fat tails for the distribution of residuals. Those not familiar with CRIX Index can visit thecrix.de or read the interesting paper written by (Trimborn and Härdle, 2016). In (Chen et al., 2018) the SVCJ (Stochastic Volatility with Correlated Jumps) model for the CRIX index and Bitcoin is implemented, obtaining interpretable jump locations and also an almost perfect fit in the distribution of residuals when comparing with a normal distribution QQ-plot, suggesting the residuals follow a Gaussian distribution.

In order to complement the previous findings, that is, the existence of an interesting family of econometric models (stochastic volatility models with jumps) for the cryptos (idea that was also previously contemplated by (Gronwald, 2014)), and the increasing interest in developing a derivatives market for cryptos, the present research was conceived. This research is practitioner oriented with 3 main objectives: The first one is to bring closer the SVCJ to practitioners, specially to those not familiar with it. The second objective is to extend some econometric analysis to cryptos different to Bitcoin, which continues to be the most important crypto in terms of market capitalization, but leave out 50% of the market. The third objective is to contribute with a grain of sand to the development of the derivates market by summarizing the first two objectives into a practical cryptocurrency option price estimation.

The three objectives are connected and will be easily accessible trough an online application that can be accessed going to https://svcjoptionpricing.shinyapps.io/optionapp/. I will do my best to keep the app always accessible but due to technological changes or external conditions of the server it could be possible that instead users will have to refer to the original code that will be always available on www.quantlet.de (users only need to enter the name that will appear next to this icon in the present text).

This paper is organized as follows: section 2 introduces the SVCJ model, presenting its main components as well as a brief reminder of bayesian estimation. Section 3 presents the methodology implemented, explaining the basic idea behind MCMC (Markov Chain Monte Carlo) and the Metropolis-Hastings algorithm, as well as, mentioning details about the aforementioned app. Section 4 is devoted to present some of the most interesting results. Section 5 concludes with some final remarks.

2 SVCJ Model

2.1 Building Blocks

A good approximation to understand the SVCJ model is starting by describing its component blocks one by one until we get the final model. In that order the initial component of the SCVJ is the Geometric Brownian motion which is described by the following formula:

$$dP_t = \mu P_t dt + \sigma P_t dW_t \tag{1}$$

In this case the price P is described by a long term trend or mean μ (also called drift) and some perturbation or diffusion given by the volatility constant σ and the Brownian (Wiener) process dW_t. An additional step could be to complement the previous model by introducing mean adjustment parameter, such as is described by the Cox-Ingersoll-Ross model (CIR) (Cox et al., 2005). The formula that describes the CIR model is the following:

$$dP_t = \kappa\{\mu - P_t\}dt + \sigma\sqrt{P_t}dW_t \tag{2}$$

In this case a new parameter κ is included to represent the speed of convergence of P_t to its mean μ. Moving forward we can think of the volatility as a stochastic process having its own equation, in that case we speak about the Stochastic Volatility Process (SV), or the Heston Model (Heston, 1993), which is given by the two following formulas:

$$dY_t = \mu dt + \sqrt{V_t}dW_t \tag{3}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v\sqrt{V_t}dW_t \tag{4}$$

Notice that the first equation of the SV defines a Geometric Brownian motion and the Volatility equation is represented by the CIR model. Also notice that the right hand side of the Geometric Brownian motion does not contain P_t anymore, in doing so we let the Y_t on the left hand side to represent returns instead of prices, i.e, $dY_t = dP_t/P_t$, which is nothing else than the definition of returns. The last step to get the SVCJ, consist in adding Jump Sizes Z_t and Jump frequencies N_t as was proposed by (Bates, 1996) in what is know as the SVJ (Stochastic Volatility with Jumps) model.

2.2 SVCJ Model description

(Duffie et al., 2000) used the SVJ model, as the base model, and introduced correlation between the jump in prices and jumps in volatility to finish with what we know as the SVCJ model. To understand the concept let dY_t be the return process and V_t the volatility process, then the SVCJ model is represented by the following equations:

$$d\log(S_t) = \mu dt + \sqrt{V_t}dW_t^{(s)} + Z_t^{(y)}dN_t \tag{5}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v\sqrt{V_t}dW_t^{(v)} + Z_t^{(v)}dN_t \tag{6}$$

$$Cov(dW_t^{(s)}, dW_t^{(v)}) = \rho dt \tag{7}$$

$$P(dN_t = 1) = \lambda dt \tag{8}$$

The parameters κ and θ are the volatility mean reversion rate and mean reversion level respectively. The parameter θ is the long run mean of V_t and the process reverts to this level at a speed governed by the parameter κ. The parameter σ_v is referred to as the volatility in volatility. $W^{(s)}$ and $W^{(v)}$ are two correlated standard Brownian motions with correlation ρ. N_t is a pure jump process with a constant mean-jump arrival rate λ, in other words, N_t represents the jump frequency. The random jump sizes are determined by Z_t^y and Z_t^v. The distributions of the random jump sizes are:

$$Z_t^y|Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2) \tag{9}$$

$$Z_t^v \sim Exp(\mu_v) \tag{10}$$

Notice that Z_t^v follows an exponential distribution to ensure that the jumps in volatility are positive. The empirical calibration of parameters is based on the following Euler discretization:

$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t \tag{11}$$

$$V_t = \alpha + \beta V_{t-1} + \sigma_v\sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t \tag{12}$$

Where $Y_{t+1} = \log(S_{t+1}/S_t)$ is the log return, $\alpha = \kappa\theta$, $\beta = 1-\kappa$ and ε_t^y, ε_t^v are the $N(0,1)$ variables with correlation ρ. J_t is a Bernoulli random variable with $P(J_t = 1) = \lambda$ and the jump sizes Z_t^y and Z_t^v are distributed as specified in equations 9 and 10

2.3 Bayesian Estimation

To understand how the Markov Chain Monte Carlo (MCMC) technique from section 3.1 estimates the model, some concepts of Bayesian Statistics are useful at this point.

The three basic components of Bayesian Modeling are: the likelihood, the prior distribution and the posterior distribution. The first one, the likelihood, refers to probabilistic model for the data, in other words, it describes how the data is generated given some parameters. It is usually represented by $p(Y|\Theta)$, where Θ represents the set of parameters and Y represents the dependent variable. The prior distribution is the way our parameter is distributed and is represented as $p(\Theta)$. The third component, the posterior probability, is the distribution of the parameters given the data and is represented as $p(\Theta|Y)$. We can relate the previous probabilities using the famous Bayes Theorem in the following way:

$$p(\Theta|Y) = p(Y|\Theta)p(\Theta) \tag{13}$$

In our particular case of the SVCJ model, we also have a set of covariates X (jump Size Z, volatility V and jump frequency J), so the the equation relating them will be:

$$p(\Theta, X|Y) = p(Y|\Theta, X)p(X|\Theta)p(\Theta) \tag{14}$$

We are interested on the left hand side of equation 14, that is, to get the distribution of the parameters and the covariates, given the data, but the way to solve the puzzle is to solve the right hand side of the previous equation. Appendix A summarizes some of the mathematical expressions that govern the posterior probability and were used for SVCJ estimation.

3 Methodology and Implementation

3.1 Markov Chain Monte Carlo (MCMC)

Due to the complexity of the model, some of the posterior distribution are represented by formulas that cannot be computed by traditional analytical methods. Instead, computational simulations are required in order to draw samples from those distributions.

To start conceptualizing the MCMC let's consider a sequence $X_1, X_2, X_3, ..., X_n$ of random variables. According to the law of total probability, the joint probability of the sequence can be expressed as:

$$p(X_1, X_2, ..., X_n) = p(X_1)p(X_2|X_1)p(X_3|X_2, X_1) \cdot ... \cdot p(X_n|X_{n-1}, ..., X_2, X_1) \tag{15}$$

Now, according to *Markov Property* we know that, given the entire past history of a sequence, the probability distribution of the random variable for the next step only depends on the current value. Mathematically, the *Markov Property* is defined as:

$$p(X_{n+1}|X_t, X_{t-1}, ..., X_2, X_1) = p(X_{t+1})$$

Under this assumption we can express equation 15 as:

$$p(X_1, X_2, ..., X_n) = p(X_1)p(X_2|X_1)p(X_3|X_2) \cdot ... \cdot p(X_n|X_{n-1}) \tag{16}$$

which is much simpler than original equation 15. This simplicity translates in the moment of making samples from the distributions and can be seen directly from observing the right hand side of equation 16. One can start by drawing a sample from the distribution of X_1, then use it to get the distribution of X_2 given X_1, $P(X_2|X_1)$, and continue this way. From a high level perspective this is the spirit of the *Markov Chain Monte Carlo.*

Up to some point, the previous example illustrates the way to proceed with equation 14. To start, we draw a sample from the prior distribution $p(\Theta)$, then using this information we compute $p(X|\Theta)$, and finally, once we have Θ and X, we compute the likelihood $p(Y|X, \Theta)$. Some more detail about the mathematical expressions that govern the posterior probabilities is given in the Appendix. For the moment it is important to mention that the process of drawing a sample from the prior distribution until getting the full posterior is not arbitrary and needs to follow some rules which are ruled by an algorithm. In the present case I use the Metropolis Hastings algorithm.

3.2 Metropolis-Hastings Algorithm

Metropolis Hastings algorithm has the advantage of letting us to sample from a generic probability. Let's suppose we want to pick a sample from a sample distribution $p(\Theta)$ but we do not know all the details about that distribution, we know this function p up to a function $g(\Theta)$ which is proportional to $p(\Theta)$, in this case the Metropolis Hastings algorithm will proceed as follows:

a) Select an initial value θ_0

b) For $i = 1, ..., m$ repeat:

i) Draw a candidate $\theta^* \sim q(\theta^*|\theta_{i-1})$

ii) Compute the following ratio $\alpha = \frac{g(\theta^*)q(\theta_{i-1}|\theta^*))}{g(\theta_{i-1})q(\theta^*|\theta_{i-1})}$

iii) If $\alpha \geq 1$ then accept θ^* and set $\theta_i \leftarrow \theta^*$

If $0 < \alpha < 1$ accept θ^* and set $\theta_i \leftarrow \theta^*$ with prob. α

If $0 < \alpha < 1$ reject θ^* and set $\theta_i \leftarrow \theta_{i-1}$ with prob. $1 - \alpha$

Steps ii) and iii) work as a correction because we are drawing samples from a proposal function q which is not the target distribution p. An easy way to understand the Metropolis Hasting algorithm is to think of step ii) as the ratio of two posterior probabilities, the new posterior probability over the old posterior probability. If we draw a θ^* which increases the new posterior probability over the old posterior probability, that is $\alpha \geq 1$, then we select θ^* as the new candidate for θ, if not it does not mean that we discard this θ^* but it means that we accept it with a probability α. This acceptance probability can be defined as $min(\alpha, 1)$ A common way to achieve this probability is to draw a uniform number $u \sim Uniform(0, 1)$ and then select the new candidate θ^* when $u < min(\alpha, 1)$, in that way if, for example, $\alpha = 0.3$, we know that $u < 0.3$ will occur with probability 0.3 the same as the required probability α, so we will accept θ^* with probability α whenever $u < min(\alpha, 1)$.

It is important to mention that the aforementioned algorithm depends on the starting values θ_0. We can reduce the influence of the starting values by discarding the first part of the chain. This is usually known as the *Burn in rate*. For the purposes of this work the default *burn in rate* will be around 20%, nevertheless it can be changed anytime in the code. An additional aspect of the Metropolis Hastings algorithm has to do with the autocorrelation the different values of θ. It is normal to expect some autocorrelation due to the nature of the Markov Chain but excess of autocorrelation deserve further investigation to discard model specification problems.

3.3 Option Price Estimation

The following section pretends describe the assumptions that were considered for the option pricing but also incorporating general concepts that could be used by future students or practitioners to help them better understand the topic.

The Option Price Estimation is done for European Options. A European Option can only be exercised once the option has reached the expiry date, which is denoted with the letter T. A Call Option gives the holder the right to buy at a specific price, known as exercise price and denoted by K. The Call Option pay-off is determined by the following expression:

$$C_T = \max(0, S_t - K)$$

Where S_T represents the price of the asset at maturity. Similarly, a Put Option gives the holder the right to sell an asset. The Put Option pay-off is given by the following expression:

$$P_T = \max(0, K - S_T)$$

The price of a Call Option at a time t, before expiration T, can be expressed as:

$$C(S_t, K, T, t, r, V) = \exp^{-r(T-t)} \int_0^\infty \max(0, S_t - K) q(S_T|S_t, K, T, t, r, V) dS_T \tag{17}$$

Where $q(S_T|S_t, K, T, t, r, V)$ is known as the Risk Neutral Density (RND), which depends on K, T, r but also on the volatility V. Equation 17 is nothing else than taking the expectations of the Call Option pay-offs using a RND and then discounting it by the factor $\exp^{-r(T-t)}$. Moreover, following the results of (Breeden and Litzenberger, 1978), we know that the the RND can be recovered in the following way:

$$q(S_T|S_t, K, T, t, r, V) = \exp^{r(T-t)} \frac{\partial^2 C(S_t, K, T, t, r, V)}{\partial K^2}$$

In practice the real volatility V is not observable and it is replaced by its observable counterpart Z and also one have to evaluate the previous equation with respect to the moneyness $m = \frac{K}{S_t}$ as follows:

$$q(S_T|S_t, K, T, t, Z) = \frac{1}{S_t} \exp^{r(T-t)} \frac{\partial^2 \overline{C}}{\partial K^2}$$

Where $\overline{C}(m, T, t, Z) = \frac{1}{S_t} C(K, T, t, Z, S_t)$. One further simplification of the problem consists in using a finite difference approximation such as the following, see (Hull and Basu, 2016):

$$\frac{\partial^2 C(\cdot)}{\partial K^2} \approx \frac{C(K = S - \Delta K, T) - 2C(K = S, T) + C(K = S + \Delta K, T)}{(\Delta K)^2}$$

There are several techniques that let us recover the Option Call Price given some explanatory variables (T, t, m, Z). Between them the non-parametric techniques are widely used, see (Belomestny et al., 2015) or (Lykhnenko, 2016). Nevertheless for the case of cryptos there are no real Call Options prices and therefore to fit a model, parametric or not, imposes some difficulties. A different approach could be to try to link the RND to a real-world probability. Fortunately there is a way to link the two measures by using the Girsanov Theorem, see (Girsanov, 1960). Let's define define the quantity λ as the excess of return over the risk-free rate:

$$\lambda = \frac{\mu - r}{\sigma}$$

Then, by means of Girsanov Theorem we have the following relation:

$$C(S_t, K, T, t, r) = \mathbb{E}^{\mathbb{Q}}[\exp^{-r(T-t)} \phi(S_t)] = \mathbb{E}^{\mathbb{P}}[\exp^{-(r + \frac{\lambda^2}{2} + \frac{\lambda}{T} W_t{}^{\mathbb{P}})T} \phi(S_t)] \tag{18}$$

Where $\phi(S_t)$ is the Call Option pay-off. Under equation 18 we can use the usual expectation $\mathbb{E}^{\mathbb{P}}$ and the exercise reduces to adjust the discount parameters to include the risk premium λ. For the the present thesis I assume this risk premium to be 0. This could be debatable, but since there are no real options prices it is one way to avoid further assumptions that could also be debatable. Additionally, as I will present in section 4, the long term return μ for several cryptos is between 0.02 and 0.03 which is also very close to the 3 months Treasury Bond rate, the proxy of r.

3.4 Shiny Application

The application can be access through https://svcjoptionpricing.shinyapps.io/optionapp/. It was done using R version 3.5.1 (Team, 2018), using the packages shiny (Chang et al., 2018) and shinydashboard (Chang and Borges Ribeiro, 2018). All the codes and important references can be found in the reference section of the app.

Once the app is launched users will see a welcome message that briefly explains the purpose of the app. First step consist on clicking "Step 1- Select Crypto" on the left hand panel. There the constitutes of the CRIX index will appear and users will be able to select the crypto for which they want to calculate the option prices. In the next step called "Step 2 - SVCJ Parameters", users will see the already calibrated parameters of the SVCJ model for every crypto on the left hand side. The SVCJ calibrated parameters are an input for the app and can be found on the code repository as well. On the right hand side a brief explanation of the SVCJ model is presented. The parameters on the left hand side are editable but the price simulation will only consider the default parameters so it does not matter if the user inserts different values for the parameters.

Third step called "Step 3 - Option Type" let users select the option type (Call or Put Option), also allowing them to select the annual free risk interest rate r. The initial or reference price of the crypto will also appear so users can decide its value (the default value is the mean price of the respective crypto). Based on this initial price the option strike price K will be selected and thereafter the option pay-offs. On the right hand side users will see a brief reminder of the option price concepts.

On the left hand side of the final step, called "Step 4 - Start Simulation", a start simulation button will let users start the simulation process. The simulation process should not take too much time since it is already optimized, nevertheless it could depend on the server speed. For that purpose an informative progress bar will appear indicating the percentage of simulation progress. What the app will do after clicking the simulation button is to simulate 5000 price paths, each with 1000 observations, using the SVCJ parameters. To shorten computational time, data with 5000 simulated return paths is already loaded in the server, so in reality what the app is doing is simply transforming the 5000 return paths into 5000 price paths by making use of the initial or reference price defined by the user.

Once the 5000 price paths are simulated, 5000 options pay-offs are computed for every strike price K. There are 21 different strike prices K, which means that there are 105000 pay-offs paths (5000 x 21), each one containing 1000 observations. Once the pay-off paths are simulated, the mean is computed for the 21 different strike prices K and for some set τ (time to maturity), resulting in a 21 x 7 matrix of mean pay-offs. The next step on the simulation is to apply the discount factor to each element of the previous matrix, resulting in the definite option price table. Final step of the simulation consist in computing the implied volatility of the option prices using the Black-Scholes formula.

Once the simulation is complete, user can see the results on the right hand side of "Step 4". The first tab called "a) Simulated Jumps from SVCJ MCMC" presents the simulated paths of the jumps in returns and jumps in volatility for the selected cryptos. This paths came from the SVCJ calibration as part of the MCMC. They are not affected by the option price simulation. The next tab called "b) Model Residual" compares the residuals of the SVCJ with the residuals of a predefined GARCH(1,1). The decision to use a GARCH(1,1) is totally arbitrary and it obeys to have an initial baseline for comparison. Please refer to (Chen et al., 2016) for a better implementation of the GARCH family models in the context of cryptocurrencies. The comparison is done by means of a QQ-plot of the model residuals and also using the Diebold-Mariano test.

The next results tab, called "c) Option Price Table", shows the option prices for different strike prices K and time to maturities τ. Users can download the table as a csv file. In case users want to change the strike prices it can be done via the initial prices of "Step - 3", since the strike prices from the option price table ranges from 0.85 to 1.15 of the initial price (i.e. the lowest strike price is 85% of the initial price). The final tab result, called "d) Implied Volatility", includes a plot of the implied volatility of the option for different times to maturities τ and strikes prices K.

To summarize all the steps:

a) Calibrate the SVCJ parameters for the returns (already done and loaded to the server before starting the app)

b) Simulate jumps in return and jumps in volatility (already done with the SVCJ calibration) c) Simulate 5000 return paths (done with steps a and b) d) Using an initial price transform them in 5000 price paths (done by the app)

e) Compute the pay-offs for a given set of K and $\tau = T - t$ f) Average the 5000 pay-offs g) Discount the previous pay-offs to get the final option price h) Compute the implied volatility by using the Black-Scholes formula

Steps a to c are inputs for the app and are already loaded in the server, users can only modify them by going to the code. Steps c to h are done by the app. Users accessing the code directly are encourage to increase the simulated paths up to 100000 to ensure a better estimation. The decision to simulate only 5000 paths obeys merely to the restrictions in the moment of deploying the app.

3.5 Data

The prices for the different cryptos were obtained from CoinGecko.com. For the case of the CRIX Index, the price was obtained from crix.de. Total CRIX index constitutes as of 31th August 2018 were 15. Nevertheless 3 of them (Miota, TRX, XLM) were discarded due to its low price variation which makes it difficult to estimate returns. In that orther the total number of cryptos analyzed are 13 including the CRIX Index. The initial period varies, depending on when the crypto was released, but the last observation is fixed to 06.09.08.2018 for all cryptos. Prices are in USD. Table 3.5 presents the list of cryptos included with their corresponding period of analysis.

Crypto	Long Name	Initial Date	Final Date	mean price	sd price
ADA	Cardano	18.10.2017	06.09.2018	0.25	0.21
BCH	Bitcoin Cash	02.08.2017	06.09.2018	1038.8	631.11
BTC	Bitcoin	01.08.2014	06.09.2018	2130.20	3448.02
CRIX	CRIX Index	31.07.2014	06.09.2018	7486.5	11747.31
DASH	DASH	14.07.2014	06.09.2018	122.83	230.64
EOS	EOS	09.07.2017	06.09.2018	6.53	4.85
ETC	Ethereum Classic	24.07.2016	06.09.2018	12.42	10.22
ETH	Ethereum	07.08.2015	06.09.2018	214.66	289.37
LTC	Litecoin	01.08.2014	06.09.2018	30.31	55.33
ONT	Ontology	23.03.2018	06.09.2018	4.63	2.30
USDT	Tether	01.04.2018	06.09.2018	1.00	0.02
XMR	Monero	21.05.2014	06.09.2018	51.80	92.71
XRP	Ripple	04.08.2013	06.09.2018	0.15	0.34

Table 1: List of crypptos

4 Results

4.1 Price and Returns

To start it is convenient to have a look at the prices of the CRIX Index and the CRIX Index constitues. As one can see from figure 1, the prices for the 4 cryptos with highest market capitalization, the pattern is very similar, with an almost zero price variation from the crypto release until a first moderate increase by march of 2017, continuing with a pronounced increase from october until december 2017. The maximum price for Bitcoin (BTC) was reached on 16.12.2017, for Ethereum (ETH) on 13.01.2018, for Ripple (XRP) on 07.01.2018 and for the CRIX Index on 06.01.2018. Figure 2 shows the same pattern for the cryptos with the lowest market capitalization, with the exeption of Ontology (ONT) which, since its late inception in the market, was not available by december 2017.

Moving on to analyze the returns, we can see on the figure 3 the returns for some cryptos. It is clear from figure that returns oscillate around zero with frequent changes, positive and negative. Those changes give us a visual idea of a jump. Analyzing some other returns, this time not related with market capitalization, we can observe in figure 4 how the pattern

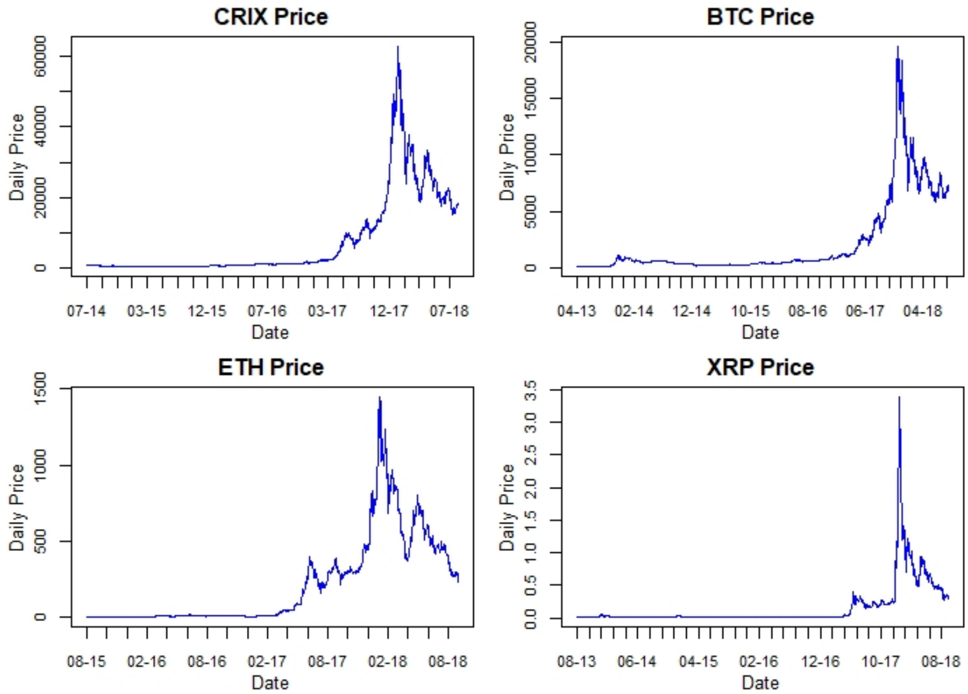

Figure 1: Price evolution in cryptos with the highest market capitalization

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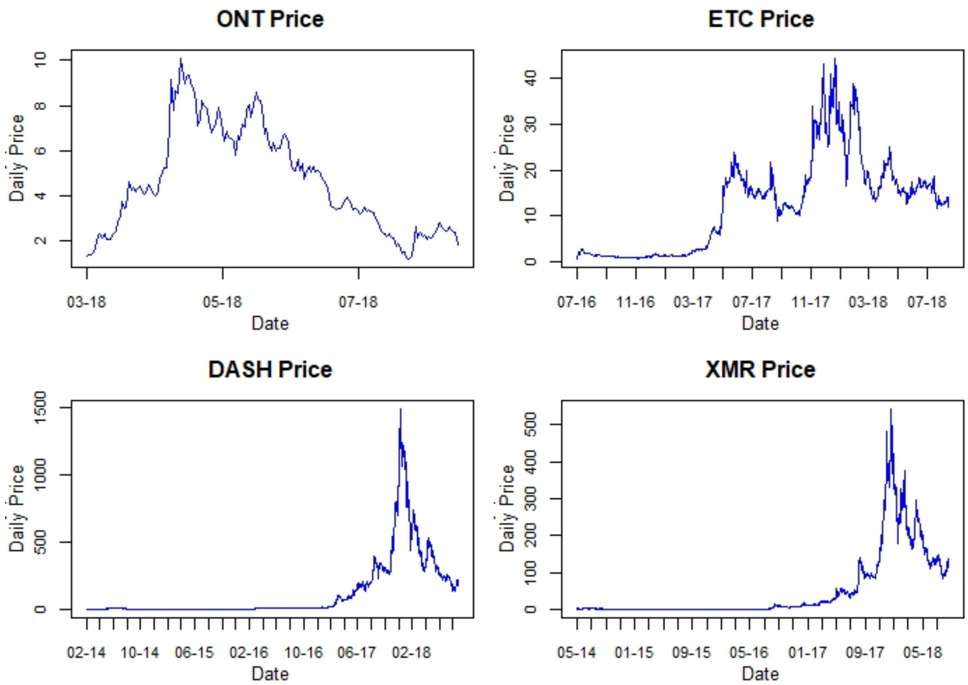

Figure 2: Daily Price (USD) in cryptos with the lowest market capitalization

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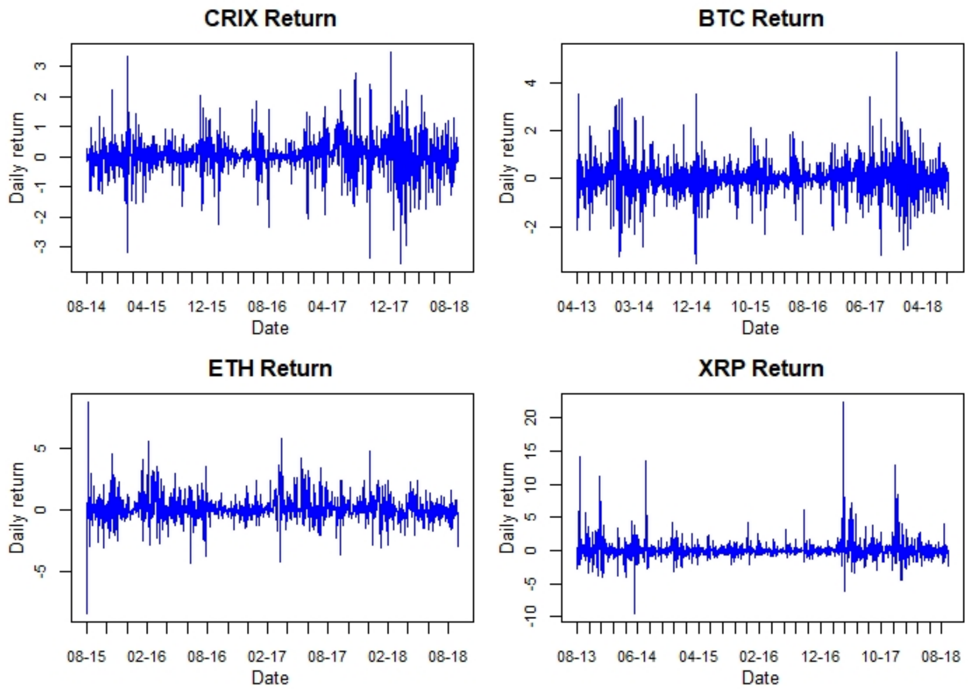

Figure 3: Returns of cryptos with highest market capitalization

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changes in some way, for example, we see how in the case of ETC there is an abrupt increase in returns at the beginning that could be related to the volatility right after its release. Other cryptos, such as ONT, present a not so dense returns pattern since the number of observations is the lowest, 168, after its release. USDT case is also striking in the way that the variation in returns is very small.

4.2 SVCJ Results

The returns previously presented were used to calculate the SVCJ model. The main code for the calculation of the SVCJ was the one used by (Chen et al., 2018) in their estimations, I did some minor changes to translate it from Matlab into R (both codes are available via www.quanlet.de). For the SVCJ estimation, a total of 5000 iterations were done in each case with a burn-in of 1000 to minimize initial value influence. Figure 5 presents the trace plot for the parameter μ (XRP not shown), which represents the long term return. It is interesting to see how the parameter fluctuates over a value near to zero in a stable interval and also not changing considerably in relation with the initial values, which supports the idea of using an

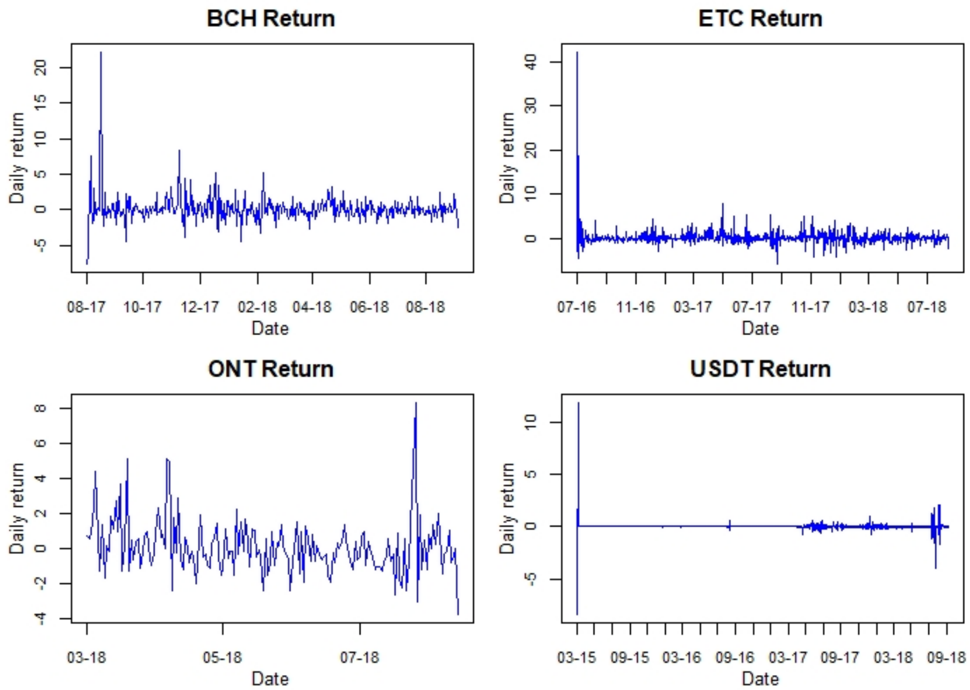

Figure 4: Returns of Bitcoin Cash (BCH), Ethereum Classic (ETH), Ontology (ONT) and Theter (USDT)

initial value of $\mu \sim N(0, 25)$ as originally proposed by (Chen et al., 2018). The case of USDT is interesting since, as shown in figure 2, the returns do not present important variations and that could explain the almost null variation of the parameter after iteration 3500.

Another interesting parameter is λ, that represents the jump arrival rate, and whose trace plot is presented in figure 6. As can be seen from the figure, λ differs between cryptos, some with higher values than others. The case of ETH seems to be interesting in the way that let us see the Metropolis-Hastings in action when the parameters is back on its path around iteration 2300. Going forward, Figure 7 presents the trace plot for the parameter ρ, correlation between Brownian motions of returns and volatility, reaching to similar conclusions as in the previous two trace plots. The remaining trace plots can be found on the code. The shiny application will not show any trace plot since it is oriented to option price estimation

Table 2 shows the estimated parameters for the different cryptos. We can see a low MSE indicating an overall good fitting. One potential pitfall of the MCMC is that results depend on the initial values of the parameters. As already mentioned one possible way to solve that problem is to drop a certain number of initial estimates in what is known as the burn-in

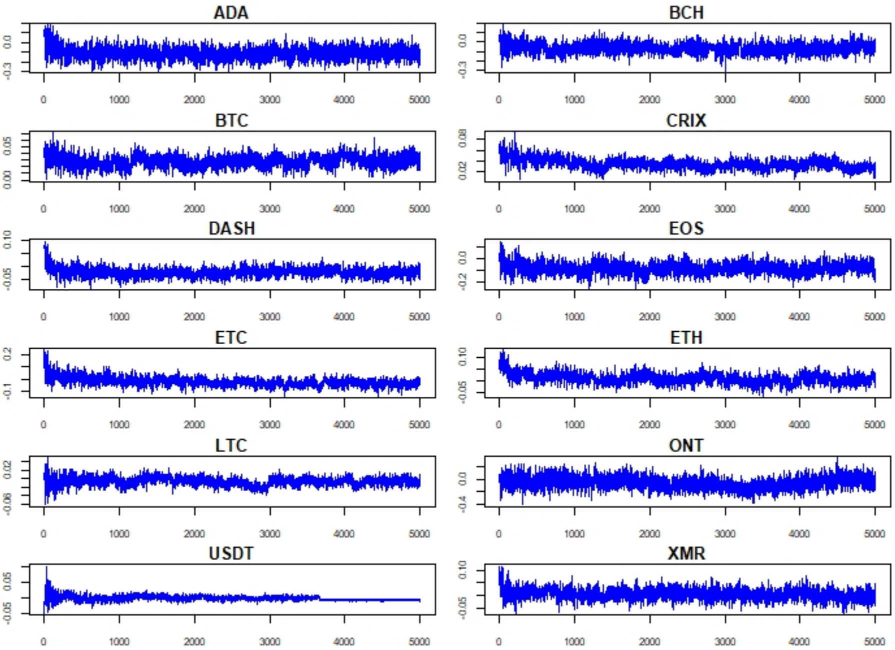

Figure 5: Trace Plot of parameter μ (MCMC iteration on x-axis)

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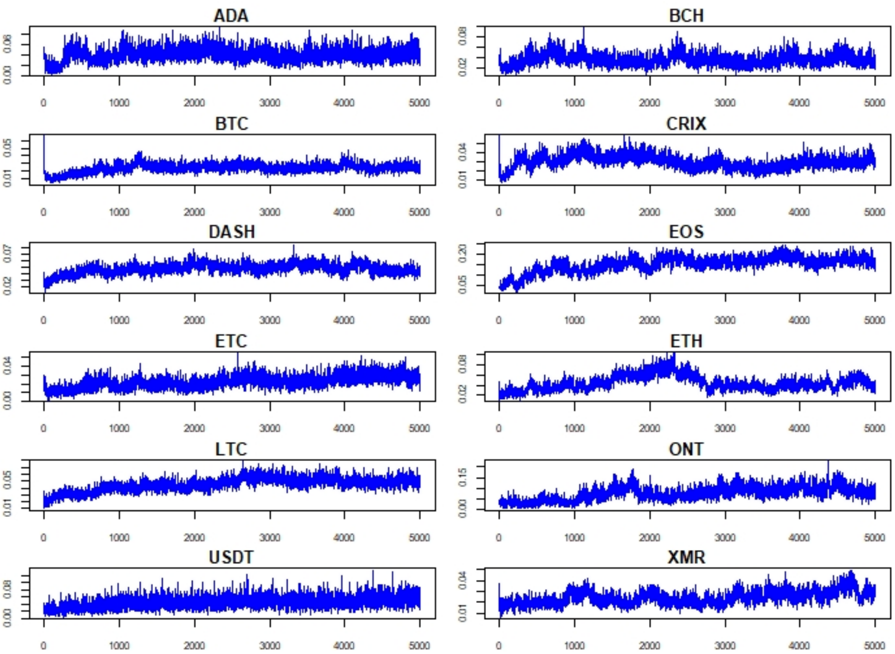

Figure 6: Trace Plot of parameter λ (MCMC iteration on x-axis)

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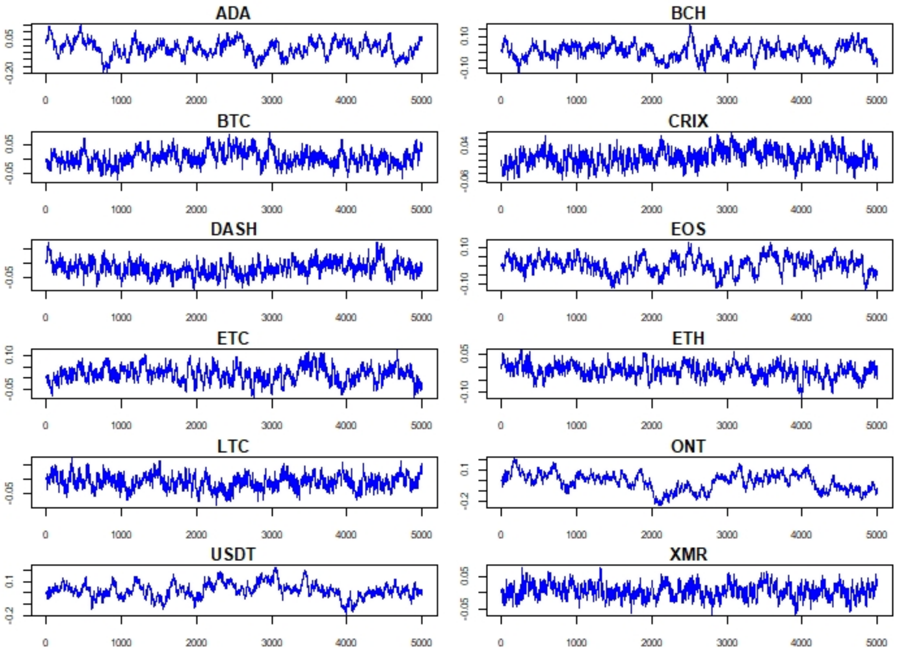

Figure 7: Trace Plot of parameter ρ (MCMC iteration on x-axis)

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period. Still, even using a considerable burn-in there could be changes in the results depending on the initial values. An additional way to prevent this is to use initial values that one extract observing the data. Unfortunately this is not easy and, eventually, only applies for few parameters such as μ or σ_v. For the rest of the parameters, to infer the initial values by observing the data could be more complicated.

Table 3, shows a simple sensitivity analysis for the parameter mu of BTC where the estimates are presented for different initial values. The decision to take an initial value for μ following a Normal distribution with mean $a = 0$ and standard deviation $A = 25$ is coming for the posterior equation presented in section A.1 in one hand, and from the observed long term trend of the return which oscillates around 0 for every crypto, on the other hand. The discussion could be centered in the initial value of the standard deviation A and that is what is presented in table 3.

Lets take now the example of the parameter σ_y, again for BTC, which represents the standard deviation of the jump size Z_t^y. The parameter σ_y follows an Inverse Gaussian dis-

Crypto		μ	μ_y	σ_y	λ	α	β	ρ	σ_v	ρ_j	μ_v	MSE
ADA	mean	-0.115	0.006	52.5	0.043	0.364	-0.418	-0.022	0.045	0.039	6.425	0.897
	sd	0.057	0.740	23.176	0.012	0.097	0.106	0.054	0.027	0.085	2.141	
BCH	mean	-0.066	-0.046	59.484	0.035	0.257	-0.273	0.013	0.090	0.027	5.532	0.816
	sd	0.052	0.713	27.868	0.011	0.069	0.055	0.048	0.032	0.095	2.307	
BTC	mean	0.029	0.004	3.974	0.025	0.009	-0.066	0.008	0.011	-0.003	0.706	0.84
	sd	0.008	0.093	1.015	0.005	0.002	0.012	0.026	0.002	0.086	0.121	
CRIX	mean	0.032	-0.002	3.145	0.030	0.010	-0.097	0.012	0.013	-0.007	0.819	0.854
	sd	0.008	0.082	0.755	0.007	0.003	0.018	0.024	0.002	0.069	0.181	
DASH	mean	-0.023	0.100	17.345	0.049	0.049	-0.161	-0.019	0.032	-0.003	2.129	0.866
	sd	0.015	0.187	4.008	0.007	0.008	0.017	0.025	0.005	0.059	0.265	
EOS	mean	-0.082	0.081	8.201	0.164	0.161	-0.450	-0.002	0.032	0.013	2.203	0.743
	sd	0.044	0.248	1.680	0.027	0.030	0.087	0.048	0.019	0.075	0.425	
ETC	mean	-0.031	-0.479	99.434	0.024	0.221	-0.326	0.018	0.118	0.052	12.541	0.779
	sd	0.030	0.620	32.063	0.007	0.094	0.088	0.037	0.028	0.037	3.609	
ETH	mean	0.007	0.011	4.650	0.047	0.029	-0.094	-0.018	0.032	0.000	1.594	0.886
	sd	0.019	0.120	1.501	0.014	0.008	0.011	0.027	0.007	0.051	0.393	
LTC	mean	-0.007	0.005	8.177	0.048	0.011	-0.123	-0.012	0.013	0.026	1.391	0.852
	sd	0.009	0.136	1.694	0.008	0.003	0.010	0.025	0.004	0.065	0.211	
ONT	mean	-0.067	0.079	10.554	0.085	0.505	-0.565	-0.029	0.033	0.013	3.228	1.026
	sd	0.093	0.458	4.391	0.028	0.164	0.111	0.080	0.020	0.099	1.038	
USDT	mean	-0.001	-0.032	7.536	0.048	0.003	-0.386	0.024	0.004	0.024	1.278	0.389
	sd	0.006	0.395	3.775	0.016	0.001	0.036	0.069	0.001	0.232	0.405	
XMR	mean	0.003	0.038	29.024	0.025	0.063	-0.098	0.004	0.048	0.007	2.032	0.869
	sd	0.021	0.247	9.691	0.006	0.012	0.018	0.022	0.008	0.081	0.369	
XRP	mean	-0.043	0.054	20.358	0.043	0.026	-0.156	-0.018	0.025	0.002	3.095	0.834
	sd	0.012	0.185	4.127	0.006	0.005	0.011	0.022	0.006	0.040	0.564	

Table 2: SVCJ estimated parameters

Parameter				
μ	Init. Val:	$a = 0, A = 10$	$a = 0, A = 25$	$a = 0, A = 100$
	Estimate:	0.030	0.029	0.033
σ_y	Init. Val:	$f = 5, F = 40$	$f = 100, F = 40$	$f = 5, F = 200$
	Estimate:	3.974	0.540	20.466

Table 3: BTC μ and σ_y parameters for different initial values

tribution. A couple of different initial values were used in table 3, but unlike μ, their choice is more difficult to support and without previous information they are completely arbitrary. We can see from the table how the parameter estimates differs considerably. Figure 8 also facilitates the visualization while showing different trace plots for the parameter σ_y, according to different initial values. We can see on the Y-axis how the values differ, even after the burn-in period.

Table 3 is by no means a complete sensitivity analysis because the range of initial values to be tested needs to be enlarged considerably. Even, expanding the grid of potential initial values, results could be misleading if we omit important prior knowledge. A more sophisticated numerical technique should be applied, requiring considerable computing time, but is beyond the scope of this paper. The objective of the table is only to warn about the shortcomings of the MCMC in the context of SVCJ estimation, by presenting a very simple example. A good reference to improve the results can be found in (Kristensen and Shin, 2012), where a non parametric simulated maximum likelihood is used for dynamics models where no closed-form representation of the likelihood function is available providing hints when considering an alternative way to set up the initial parameters.

Nontheless, an interesting property of the MCMC is that, as its name indicates, it works as a chain, with the advantage of estimating not just parameters but also volatility and covariates. Figure 9 shows the estimated volatility under the SVCJ. It is interesting to see the increased volatility period at the end of year 2017. We can see how the estimated volatility is higher for XRP compared with CRIX or ETH, for example. As in the case of the parameters

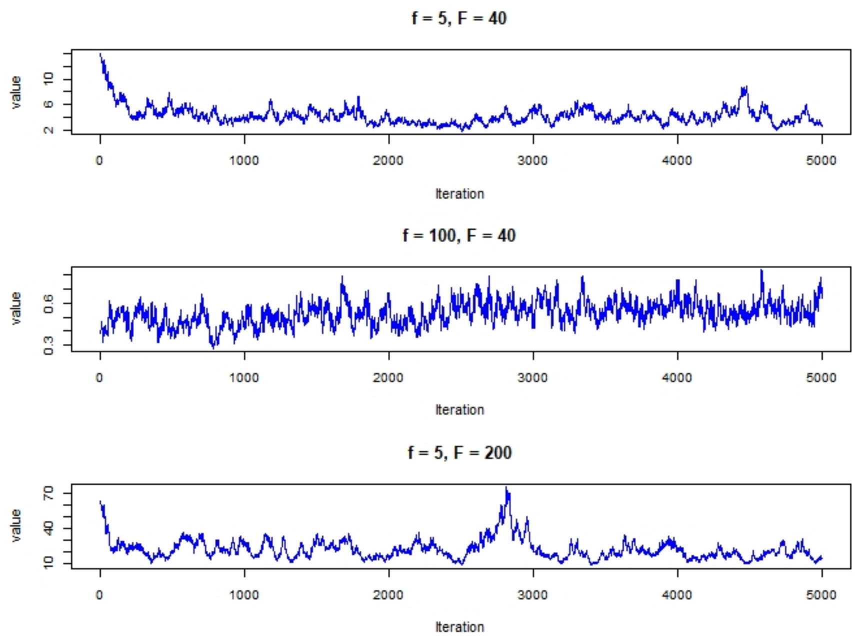

Figure 8: Trace plots for parameter σ_y of BTC using different initial values f and F (MCMC iteration on x-axis)

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trace plots, the estimated SVCJ volatility plot is not part of the app results. Fortunately for the users all the following plots can be also accessed using the app.

To start with the app results, another interesting plots coming from the SVCJ model are the estimated jumps in returns and volatility, presented in figure 10 where we can see the jumps in returns on the left column and the jumps in volatility on the right column. We can see how, compared to BTC, BCH exhibits a lower frequency of jumps. On the contrary, DASH highlights for having a high frequency of jumps. XMR is also interesting since the size of positive jumps in returns is higher compared to the negative jumps. In neither case the jumps in volatility are negative due to the model definition which set them up following an exponential distribution. A more detailed plot of jumps can be seen using the app since only one crypto at a time is plotted there.

The second result users can find on the app is the QQ-plot of the SVCJ residuals, such as those plotted in figure 11. It is clear from the figure how the SVCJ model residuals seems to follow a normal distribution which speaks about a good model fitting. The residuals follow

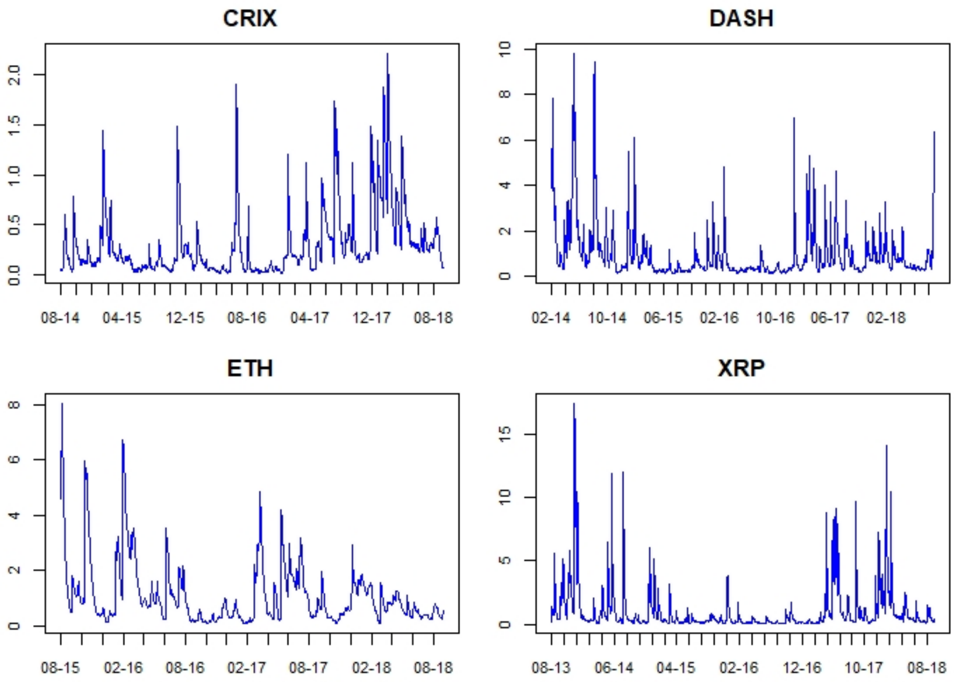

Figure 9: SVCJ in-sample fitted volatility

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the pattern mentioned by almost all the cryptos. One exception, which is shown on purpose on the lower-right panel of figure 11, is USDT. We can see how the residuals deviate from the red line, indicating that they possibly do not follow a normal distribution. Additional comparisons can be done by users on the app, such as comparing the SCVJ residuals with a GARCH (2,2) model residuals. For a more deep analysis of the econometrics of CRIX and BTC please refer to (Chen et al., 2016).

Once we have the SVCJ parameters next step consist in simulate 5000 returns paths for each crypto. Figure 12 shows two simulated returns paths for ETH. The simulated paths show jumps with the desired frequency and size, also one can see how the returns oscillate around the zero line. If we use a reference or initial price, we can transform the simulated returns of figure 12 into simulated prices. Such is the case shown in figure 13, were five different price paths are shown, again for the case of ETH, using an initial price of 215 USD. We can observe, as well as in the previous figure, the well defined jumps, driving prices up or down depending on the path and the observation. For every crypto 5000 price paths are simulated in the app using the SVCJ parameters.

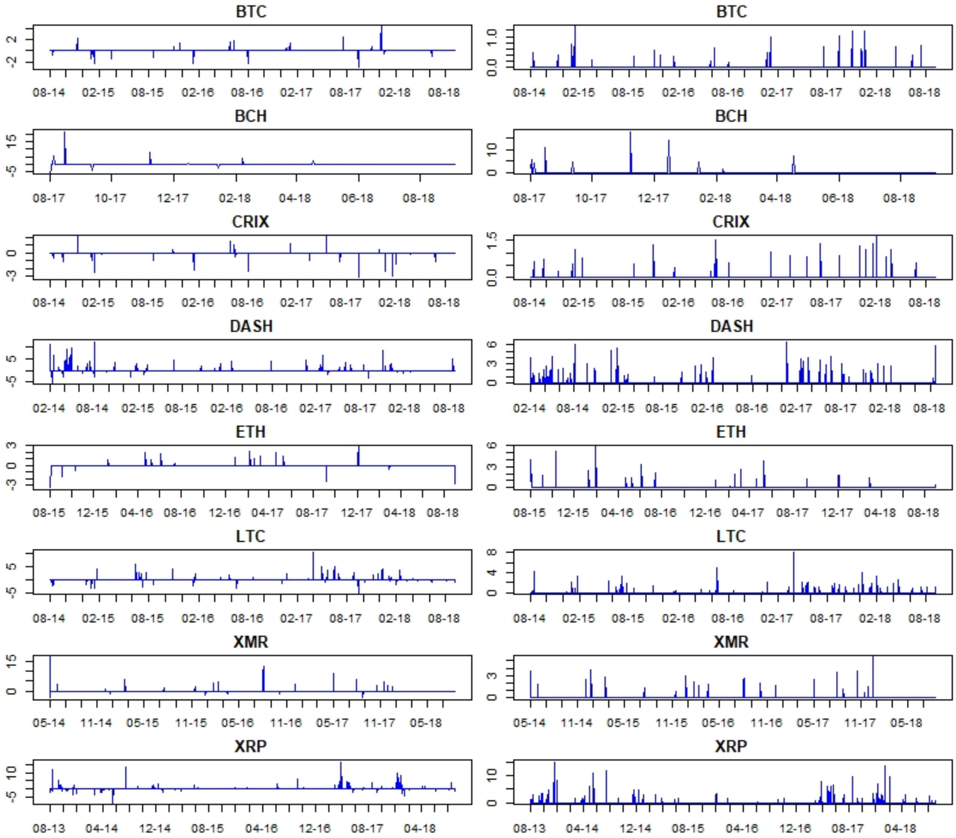

Figure 10: Estimated jumps in returns (left column) and volatility (right column)

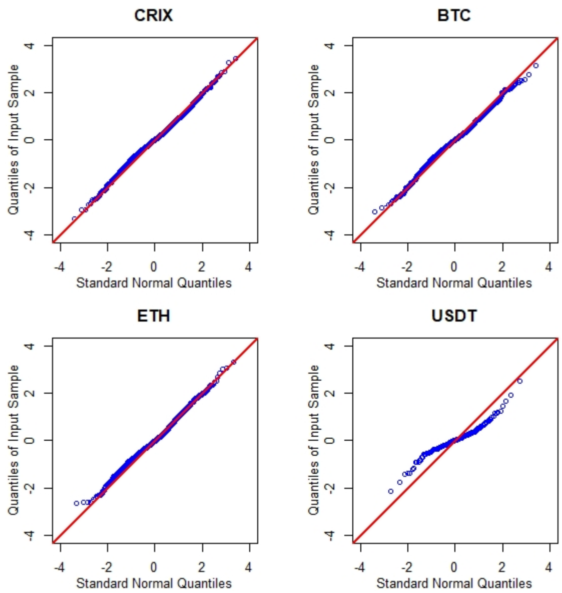

Figure 11: SVCJ residuals

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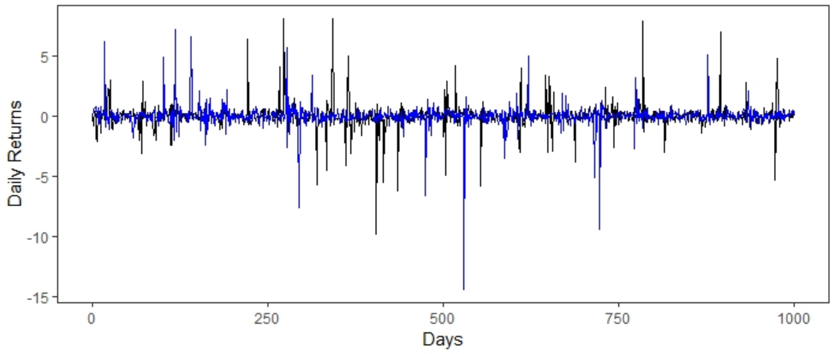

Figure 12: Two simulated return paths for ETH (one blue, one black) using SVCJ parameters

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Having the simulated price paths, what remains is to compute option prices. The way to do that is defining one strike price K and one time to maturity t and then computing the option pay-off according to formulas from section 3.3, depending if we want a call or a put option. Then to take expectations of the pay-off and discount that value. The result, again for the case of ETH, can be seen in table 4. Users can get similar tables with the app even with the possibility of download them into a csv file.

Last but not least, the final result of the app is the Implied Volatility computed using the Black Scholes formula as the one depicted in figure 14

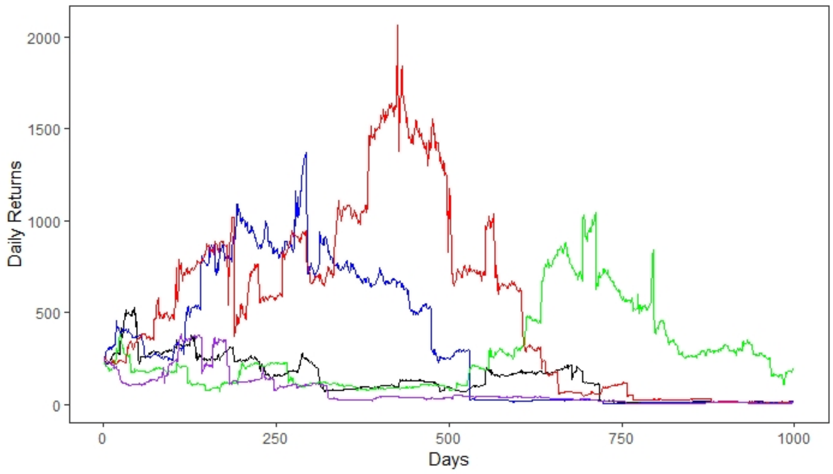

Figure 13: Five simulated price paths for ETH using an initial price of 215 USD

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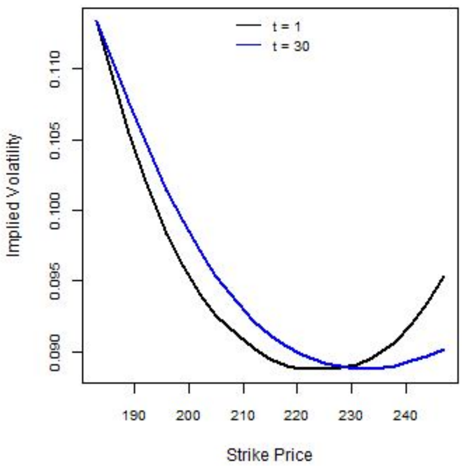

Figure 14: Implied Volatility for ETH Call Option using Black-Scholes formula

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K/t	1	7	30	60	90	180	360
183	33.17	39.05	54.82	69.85	83.39	112.68	147.72
186	30.32	36.66	52.90	68.21	81.87	111.46	146.78
189	27.51	34.34	51.01	66.60	80.38	110.25	145.86
192	24.76	32.10	49.17	65.02	78.92	109.05	144.95
196	21.23	29.24	46.79	62.97	77.01	107.49	143.75
199	18.70	27.20	45.07	61.47	75.62	106.33	142.86
202	16.30	25.26	43.41	60.01	74.25	105.19	141.99
205	14.05	23.40	41.80	58.58	72.91	104.07	141.12
209	11.34	21.07	39.74	56.71	71.16	102.59	139.97
212	9.52	19.45	38.26	55.35	69.87	101.50	139.12
215	7.89	17.92	36.83	54.02	68.61	100.43	138.28
218	6.47	16.49	35.46	52.73	67.37	99.37	137.45
221	5.25	15.16	34.14	51.47	66.16	98.33	136.63
225	3.92	13.53	32.45	49.84	64.58	96.96	135.54
228	3.12	12.41	31.25	48.65	63.42	95.94	134.75
231	2.46	11.37	30.08	47.49	62.28	94.94	133.96
234	1.93	10.42	28.97	46.37	61.17	93.96	133.18
238	1.41	9.28	27.55	44.92	59.71	92.68	132.16
241	1.13	8.52	26.54	43.87	58.63	91.74	131.41
244	0.91	7.82	25.57	42.84	57.57	90.80	130.66
247	0.74	7.20	24.64	41.85	56.53	89.88	129.92

Table 4: Call Option prices for ETH for different strike prices K and time to maturity t

5 Conclusions

Cryptocurrencies have become an important object of study for different disciplines ranging from computer science to economics, also including mathematics, statistics and law. The reasons behind that include the revolutionary technology the have brought and also the importance the cryptos started having as financial assets. We should expect more developments in the crypto market that will include the participation of more agents, that so far have seem to be reluctant, such as central and commercial banks, and also the idea of the development of a derivatives market for cryptos.

An important aspect, parallel to the development of the derivatives market, is to understand cryptos price behavior which is a challenging task since traditional econometric models, such as ARIMA and GARCH, are not necessarily the initial option to chose and more sophisticated models such as the SVCJ seem to fit the data better. Even though the SVCJ model better fit the data, there is still space for incorporating additional techniques, specially those oriented to identify the prior distributions of the parameters with their initial values.

For the case of the option price estimation, additional assumptions facilitate the estimation since there are no real option prices to compare. Those assumptions, such a zero risk premium, could be controversial but they facilitate initial exercises of option price estimation. Once real cryptos options start to be traded, more realistic assumptions can be taken in order to improve the price estimation. I hope this text could help someone interested in the crypto market to get some additional insights in how to approach the problem of returns and option price estimation. Further analysis will be required considering that cryptocurrencies came to stay.

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A Appendix

A.1 Posterior Distributions

To derive the exact formulas of the posterior distribution is beyond the scope of this paper, since they require extensive algebra and the use of conjugates distributions, nevertheless, for a complete treatment of the posterior probabilities please refer to (Numatsi, 2010). Here the only the final formulas are presented.

A.1.1 Posterior of the Parameters

- $\boldsymbol{\mu \sim N(as, As)}$

Where, for given initial values a and A:

$$As = (\frac{1}{A} + 1/(1-\rho^2)\sum_{t=1}^{T}\frac{1}{V_{t-1}})^{-1}$$

$$as = As(\frac{1}{A}a + \frac{1}{(1-\rho^2)}\sum_{t=1}^{T}W^T Q$$

$$Q = (Y_t - Z_t^y J_t - \frac{\rho}{\sqrt{\sigma_v}}(V_t - V_{t-1}(1+\beta) - \alpha - Z_t^v J_t))/\sqrt{V_{t-1}}$$

$$W = \sqrt{1/V_{t-1}}$$

- $\boldsymbol{(\alpha, \beta) \sim N(bs, Bs)}$

  Where, for given initial values b and B:

$$Bs = (\frac{1}{B} + \frac{\rho}{\sigma_v}\sum_{t=1}^{T}W^T W)^{-1}$$

$$bs = Bs(\frac{1}{B}b + \frac{\rho}{\sigma_v}\sum_{t=1}^{T}W^T Q)$$

$$Q = ((V_t - V_{t-1} - Z_t^v J_t) - \rho\sqrt{\sigma_v}(Y_t - \mu - Z_t^y J_t))/\sqrt{V_{t-1})}$$

$$W = [\frac{1}{\sqrt{V_{t-1}}}\sqrt{V_{t-1}}]$$

  $\boldsymbol{\kappa = -\alpha/\beta}$

- $\boldsymbol{\sigma_v \sim IG(cs, Cs)}$

Where, for given initial values c and C:

$$cs = c + T$$

$$Cs = C + \sum_{t=1}^{T}((V_t - V_{t-1} - \alpha - \beta V_{t-1} - Z_t^v J_t)^2 / V_{t-1})$$

- $\boldsymbol{\mu_v \sim IG(ds, Ds)}$

  Where, for given initial values d and D:

$$ds = d + 2 * T$$

$$Ds = D + 2 * \sum_{t=1}^{T} Z_t^v$$

- $\boldsymbol{\mu_y \sim N(es, Es)}$

  Where, for given initial values e and E:

$$Es = 1/(\frac{T}{\sigma_y} + \frac{1}{E})$$

$$es = Es(\sum_{t=1}^{T}(\frac{(Z_t^y - Z_t^v \rho_j)}{\sigma_y}) + \frac{e}{E})$$

- $\boldsymbol{\sigma_y \sim IG(fs, Fs)}$

  Where, for given initial values f and F:

$$fs = f + T$$

$$Fs = F + \sum_{t=1}^{T}((Z_t^y - \mu_y - \rho_j Z_t^v)^2)$$

- $\boldsymbol{\rho_j \sim N(gs, Gs)}$

  Where, for given initial values g and G:

$$Gs = (\frac{\sum_{t=1}^{T}(Z_t^v)^2}{\sigma_y} + \frac{1}{G})^{-1}$$

$$gs = Gs(\frac{\sum_{t=1}^{T}((Z_t^y - \mu_y)Z_t^v)}{\sigma_y} + \frac{g}{G})$$

- $\boldsymbol{\lambda \sim Beta(ks, Ks)}$

  Where, for given initial values k and K:

$$ks = k + \sum_{t=1}^{T} J_t$$

$$Ks = K + T - \sum_{t=1}^{T} J_t$$

A.1.2 Posterior of the Covariates

- $\boldsymbol{J_t \sim Bern(p_1/(p_1 + p_2))}$

$$p1 = \lambda \exp\Bigg(-0.5\Bigg(\frac{[Y_t - \mu - Z_t^y - (\frac{\rho}{\sqrt{\sigma_v}})(V_t - V_{t-1} - \alpha - \beta V_{t-1} - Z_t^v)]^2}{(1-\rho^2)V_{t-1}} + \frac{(V_t - V_{t-1} - \alpha - \beta V_{t-1} - Z_t^v)^2}{\sigma_v V_{t-1}}\Bigg)\Bigg) \tag{19}$$

$$
\begin{aligned}
p2 = (1-\lambda)\exp\Bigg(-0.5\Bigg(&\frac{[Y_t - \mu - \frac{\rho}{\sqrt{\sigma_v}}(V_t - V_{t-1} - \alpha - \beta V_{t-1})]^2}{(1-\rho^2)V_{t_1}} \\
&+ \frac{(V_t - V_{t-1} - \alpha - \beta V_{t-1})^2}{\sigma_v V_{t-1}}\Bigg)\Bigg) \qquad (20)
\end{aligned}
$$

Declaration of Authorship

I hereby confirm that I, Ivan Perez, have authored this master thesis independently and without use of others than the indicated sources. Where I have consulted the published work of others, in any form (e.g. ideas, equations, figures, text, tables), this is always explicitly attributed.

Berlin, October 19, 2018

Ivan Perez

Hiermit erklare ich, Ivan Perez, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgefhrten Quellen und Hilfsmittel angefertigt habe. Die Prfungsordnung ist mir bekannt. Ich habe in meinem Studienfach bisher keine Masterarbeit eingereicht bzw. diese nicht endgltig nicht bestanden.

Berlin, Oktober 19, 2018

Ivan Perez